4CE1A

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4CE1A

B. Tech. (Sem. III) MID TREM Examination,
Civil Engg.
4CE1 Strength of Materials -II

Time : 2 Hours] [Total Marks : 20

UNIT : I

Question : 1 *A beam of length 8 m is simply supported at its ends. It carries a uniformly distributed load of 40 kN/m as shown in Fig. Determine the deflection of the beam at its mid-point and also the position of maximum deflection and maximum deflection. Take E = 2×10^5 N/mm^2 and I = 4.3×10^8 mm^4.*

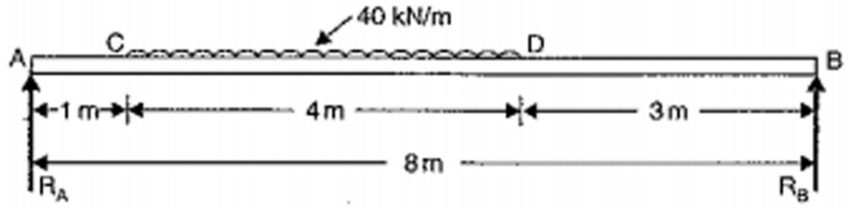


Fig.

Sol. Given :

Length, $L = 8$ m

U.d.l., $W = 40$ kN/m

Value of $E = 2 \times 10^5$ N/mm^2

Value of $I = 4.3 \times 10^8$ mm^4

First calculate the reactions R_A and R_B.

Taking moments about A, we get

$$R_B \times 8 = 40 \times 4 \times \left(1 + \frac{4}{2}\right) = 480 \text{ kN}$$

$$\therefore \quad R_B = \frac{480}{8} = 60 \text{ kN}$$

$$\therefore \quad R_A = \text{Total load} - R_B = 40 \times 4 - 60 = 100 \text{ kN}$$

In order to obtain the general expression for the bending moment at a distance x from the left end A, which will apply for all values of x, it is necessary to extend the uniformly distributed load upto the support B, compensating with an equal upward load of 40 kN/m over the span DB as shown in Fig. Now Macaulay's method can be applied.

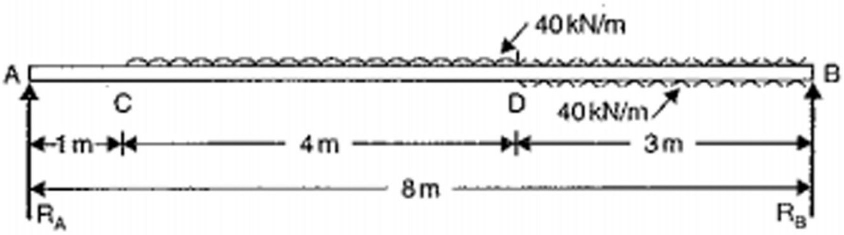


Fig.

The B.M. at any section at a distance x from end A is given by,

$$EI\frac{d^2y}{dx^2} = R_A x \;\vdots\; -40(x-1)\times\frac{(x-1)}{2} \;\vdots\; +40\times(x-5)\times\frac{(x-5)}{2}$$

or

$$EI\frac{d^2y}{dx^2} = 100x \;\vdots\; -20(x-1)^2 \;\vdots\; +20(x-5)^2$$

Integrating the above equation, we get

$$EI\frac{dy}{dx} = \frac{100x^2}{2} + C_1 \;\vdots\; -\frac{20(x-1)^3}{3} \;\vdots\; +20\frac{(x-5)^3}{3} \qquad \text{...(i)}$$

Integrating again, we get

$$EIy = 50\frac{x^3}{3} + C_1x + C_2 \;\vdots\; -\frac{20}{3}\frac{(n-1)^4}{4} \;\vdots\; +\frac{20}{3}\frac{(x-5)^4}{3}$$

$$= 50\frac{x^3}{3} + C_1x + C_2 \;\vdots\; -\frac{5}{3}(x-1)^4 \;\vdots\; +\frac{5}{3}(x-5)^4 \qquad \text{...(ii)}$$

where C_1 and C_2 are constants of integration. Their values are obtained from boundary conditions which are :

(*i*) at $x = 0, y = 0$ and (*ii*) at $x = 8$ m, $y = 0$

(*i*) Substituting $x = 0$ and $y = 0$ in equation (*ii*) upto first dotted line (as $x = 0$ lies in the first part AC of the beam), we get

$$0 = 0 + C_1 \times 0 + C_2 \qquad \therefore C_2 = 0$$

(*ii*) Substituting $x = 8$ and $y = 0$ in complete equation (*ii*) (as point $x = 8$ lies in the last part DB of the beam), we get

$$0 = \frac{50}{3}\times 8^3 + C_1\times 8 + 0 - \frac{5}{3}(8-1)^4 + \frac{5}{3}(8-5)^4 \qquad (\because \; C_2 = 0)$$

$$= 8533.33 + 8C_1 - 4001.66 + 135$$

or

$$8C_1 = -4666.67$$

or

$$C_1 = \frac{-4666.67}{8} = -583.33$$

Substituting the value of C_1 and C_2 in equation (*ii*), we get

$$EI_y = \frac{50}{3}x^3 - 583.33x \;\vdots\; -\frac{5}{3}(x-1)^4 \;\vdots\; +\frac{5}{3}(x-4)^4 \qquad \text{...(iii)}$$

(*a*) *Deflection at the centre*

By substituting $x = 4$ m in equation (*iii*) upto second dotted line, we get the deflection at the centre. [The point $x = 4$ lies in the second part (*i.e.*, CD) of the beam].

$$\therefore \quad EI.y = \frac{50}{3}\times 4^3 - 583.33\times 4 - \frac{5}{3}(4-1)^4$$

$$= 1066.66 - 2333.32 - 135 = -1401.66 \text{ kNm}^3$$

$$= -1401.66 \times 1000 \text{ Nm}^3$$

$$= -1401.66 \times 1000 \times 10^9 \text{ Nmm}^3$$

$$= -1401.66 \times 10^{12} \text{ Nmm}^3$$

$$\therefore \quad y = \frac{-1401.66\times 10^{12}}{EI} = \frac{-1401.66\times 10^{12}}{2\times 10^5 \times 4.5 \times 10^8}$$

$= -$ **16.29 mm downward. Ans.**

(*b*) *Position of maximum deflection*

The maximum deflection is likely to lie between C and D. For maximum deflection the slope $\frac{dy}{dx}$ should be zero. Hence equating the slope given by equation (*i*) upto second dotted line to zero, we get

$$0 = 100\,\frac{x^2}{2} + C_1 - \frac{20}{3}\,(x-1)^3$$

$$0 = 50x^2 - 583.33 - 6.667(x-1)^3 \qquad ...(iv)$$

The above equation is solved by trial and error method.

Let $x = 1$, then R.H.S. of equation (*iv*)

$= 50 - 583.33 - 6.667 \times 0 = -533.33$

Let $x = 2$, then R.H.S. $= 50 \times 4 - 583.33 - 6.667 \times 1 = -390.00$

Let $x = 3$, then R.H.S. $= 50 \times 9 - 583.33 - 6.667 \times 8 = -136.69$

Let $x = 4$, then R.H.S. $= 50 \times 16 - 583.33 - 6.667 \times 27 = +36.58$

In equation (*iv*), when $x = 3$ then R.H.S. is negative but when $x = 4$ then R.H.S. is positive. Hence exact value of x lies between 3 and 4.

Let $x = 3.82$, then R.H.S. $= 50 \times 3.82 - 583.33 - 6.667\,(3.82 - 1)^3$

$= 729.63 - 583.33 - 149.51 = -3.22$

Let $x = 3.83$, then R.H.S. $= 50 \times 3.83^2 - 583.33 = 6.667\,(3.83 - 1)^3$

$= 733.445 - 583.33 - 151.1 = -0.99$

The R.H.S. is approximately zero in comparison to the three terms (*i.e.*, 733.445, 583.33 and 151.1).

$\therefore$ Value of x = **3.83. Ans.**

Hence maximum deflection will be at a distance of 3.83 m from support A.

(c) *Maximum deflection*

Substituting $x = 3.83$ m in equation (*iii*) upto second dotted line, we get the maximum deflection [the point $x = 3.83$ lies in the second part *i.e.*, CD of the beam.]

$$\therefore \quad EI.y_{max} = \frac{50}{3} \times 3.83^3 - 583.33 \times 3.83 - \frac{5}{3}\,(3.83 - 1)^4$$

$$= 936.36 - 2234.15 - 106.9 = -1404.69 \text{ kNm}^3$$

$$= -1404.69 \times 10^{12} \text{ Nmm}^3$$

$$\therefore \quad y_{max} = \frac{-1404.69 \times 10^{12}}{2 \times 10^5 \times 4.3 \times 10^8} = \mathbf{-16.33\ mm.}\quad \textbf{Ans.}$$

UNIT : II

Question : 2 *A cantilever of length 3 m is carrying a point load of 50 kN at a distance of 2 m from the fixed end. If I = 10^8 mm^4 and E = 2×10^5 N/mm^2, find (i) slope at the free end and (ii) deflection at the free end.*

Sol. Given :

Length, $L = 3 \text{ m} = 3000 \text{ mm}$

Point load, $W = 50 \text{ kN} = 50000 \text{ N}$

Distance between the load and the fixed end,

$a = 2 \text{ m} = 2000 \text{ mm}$

M.O.I., $I = 10^8 \text{ mm}^4$

Value of $E = 2 \times 10^5 \text{ N/mm}^2$

(*i*) Slope at the free end is given by equation

$$\theta_B = \frac{Wa^2}{2EI} = \frac{50000 \times 2000^2}{2 \times 2 \times 10^5 \times 10^8} = \mathbf{0.005\ rad.}\quad \textbf{Ans.}$$

(*ii*) Deflection at the free end is given by equation

$$y_B = \frac{Wa^3}{3EI} + \frac{Wa^2}{2EI}(L-a)$$

$$= \frac{50000 \times 2000^3}{3 \times 2 \times 10^5 \times 10^8} + \frac{50000 \times 2000^2}{2 \times 2 \times 10^5 \times 10^8}(3000 - 2000)$$

$= 6.67 + 5.0 =$ **11.67 mm. Ans.**

UNIT : III

Question : 3 *A fixed beam AB of length 3 m carries a point load of 45 kN at a distance of 2 m from A. If the flexural rigidity (i.e., EI) of the beam is 1 × 10⁴ kNm², determine :*

(*i*) *Fixed end moments at A and B,*

(*ii*) *Deflection under the load,*

(*iii*) *Maximum deflection, and*

(*iv*) *Position of maximum deflection.*

Sol. Given :

Length, $L = 3$ m

Point load, $W = 45$ kN

Flexural rigidity, $EI = 1 \times 10^4$ kNm^2

Distance of load from A,

$a = 2$ m

∴ Distance of load from B,

$b = 1$ m

Let M_A and M_B = Fixed end moments,

y_c = Deflection under the load

y_{max} = Maximum deflection and

x = Distance of maximum deflection from A.

(*i*) The fixed end moments at A and B are given by

$$M_A = \frac{W.a.b^2}{L^2} = \frac{45 \times 2 \times 1^2}{3^2} = \textbf{10 kNm.} \quad \textbf{Ans.}$$

and

$$M_B = \frac{W.a^2.b}{L^2} = \frac{45 \times 2^2 \times 1}{3^2} = \textbf{20 kNm.} \quad \textbf{Ans.}$$

(*ii*) Deflection under load is given by equation

$$y_c = -\frac{W.a^3.b^3}{3EIL^3} = -\frac{45 \times 2^3 \times 1^3}{3 \times 1 \times 10^4 \times 3^3} = -0.000444 \text{ m}$$

$=$ **– 0.444 mm. Ans.**

–ve sign means the deflection is downwards.

(*iii*) Maximum deflection is given by equation

$$y_{max} = -\frac{2}{3EI} \times \frac{Wa^3.b^2}{(b+3a)^2}$$

$$= -\frac{2}{3 \times 1 \times 10^4} \cdot \frac{45 \times 2^3 \times 1^2}{(1+3\times 2)^2} = -\frac{16 \times 45}{3 \times 10^4 \times 49}$$

$= -0.00049$ m $= -$**0.49 m. Ans.**

(*iv*) The distance of maximum deflection from point A is given by equation

$$x = \frac{2a \cdot L}{(b+3a)}$$

$$= \frac{2 \times 2 \times 3}{1 + 3 \times 2} = \frac{12}{7} = \textbf{1.714 m.} \quad \textbf{Ans.}$$

Alternate Method

Fig. shows the simply supported beam with vertical load of 45 kN at a distance 2 m from A.

The reactions $R_A{}^*$ and $R_B{}^*$ due to vertical load will be :

$3R_B{}^* = 45 \times 2$ or $R_B{}^* = 90/3 = 30$ kN and $R_A{}^* = 45 - 30 = 15$ kN.

Fig. shows the B.M. diagram with max. B.M.at C and equal to $R_A{}^* \times 2 = 15 \times 2 = 30$ kNm.

Fig. shows the fixed beam with end moments and reactions. As the vertical load is not acting symmetrically, therefore M_A and M_B will be different. In this case M_B will be more than M_A, as load is nearer to point B. The B.M. diagram is shown in Fig.

(*i*) *Fixed end moments at A and B.* To find the value of M_A and M_B, equate the areas of two B.M. diagrams.

∴ Area of B.M. diagram due to vertical loads

= Area of B.M. diagram due to end moments

∴ $A_1 + A_2 = A_3 + A_4$ where $A_1 = \frac{30 \times 2}{2} = 30$, $A_2 = \frac{30 \times 1}{2} = 15$

$A_3 = 3M_A$, $A_4 = \frac{(M_B - M_A) \times 3}{2}$

$= 1.5\,(M_B - M_A)$

or $30 + 15 = 3M_A + 1.5M_B - 1.5\,M_A$

or $45 = 1.5M_A + 1.5M_B$

or $\frac{45}{1.5} = M_A + M_B$ or $M_A + M_B = 30$ ...(*i*)

Now equating the distance of C.G. of B.M. diagram due to vertical load to the distance of C.G. of B.M. diagram due to end moments from the some end (*i.e.*, from end A)

or $\bar{x} = \bar{x}'$

or $$\frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{A_3 \times x_3 + A_4x_4}{A_3 + A_4}$$

or $$\frac{30 \times \frac{4}{3} + 15 \times \left(2 + \frac{1}{3}\right)}{30 + 15} = \frac{3M_A \times \frac{3}{2} + 1.5\,(M_B - M_A) \times 2}{3M_A + 1.5M_B - 1.5M_A}$$

or $$\frac{40 + 35}{45} = \frac{4.5M_A + 3M_B - 3M_A}{1.5M_A + 1.5\,M_B} = \frac{1.5M_A + 3M_B}{1.5M_A + 1.5M_B}$$

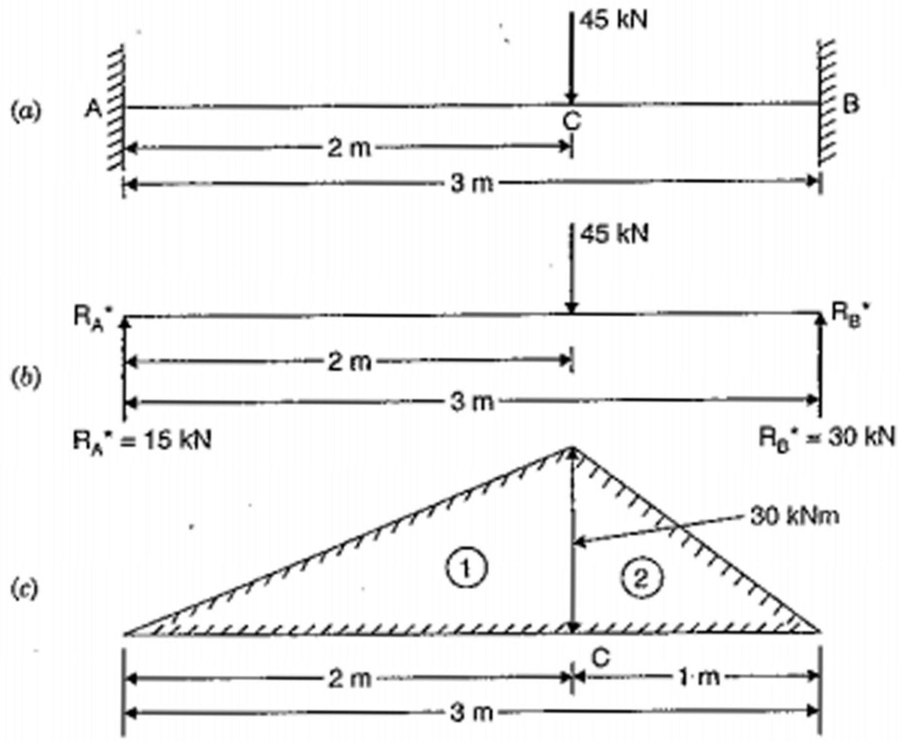
or $$\frac{75}{45} = \frac{1.5(M_A + 2M_B)}{1.5(M_A + M_B)} \quad \text{or} \quad \frac{5}{3} = \frac{M_A + 2M_B}{M_A + M_B}$$

or $$5M_A + 5M_B = 3M_A + 6M_B$$

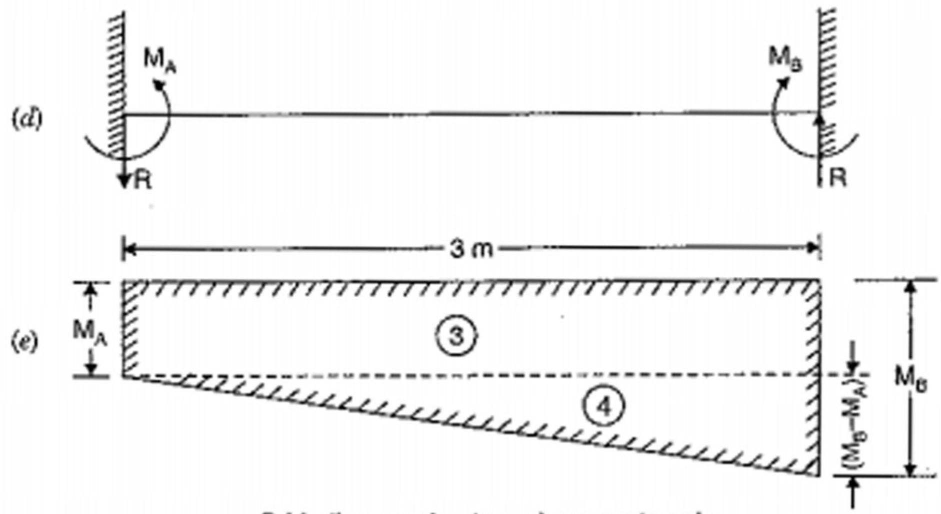
or $$2M_A = M_B \qquad \text{...}(ii)$$

Solving equations (*i*) and (*ii*), we get

$$M_A = \mathbf{10\ kNm\ and}\ M_B = \mathbf{20\ kNm.} \quad \textbf{Ans.}$$



B.M. diagram for simply supported beam with vertical loads



B.M. diagram due to end moments only

Fig.

Let us now find the reaction R due to end moments only. As the end moments are different, hence there will be reaction at A and B. Both the reactions will be equal and opposite in direction, as there is no vertical load, when we consider end moments only. As M_B is more, the reaction R will be upwards at B and downwards at A as shown in Fig.

Taking the moments about A for Fig.   we get clockwise moment at A = Anti-clockwise moments at A

$$M_B = M_A + R \times 3$$

$$\therefore \quad R = \frac{M_B - M_A}{3} = \frac{20-10}{3} = \frac{10}{3}\,\text{kN}$$

Now the total reaction at A and B will be,

$$R_A = R_A{}^* - R = 15 - \frac{10}{3} = \frac{35}{3}\,\text{kN}$$

and

$$R_B = R_B{}^* + R = 30 + \frac{10}{3} = \frac{100}{3}\,\text{kN}$$

Now, consider the fixed beam as shown in Fig.

The B.M. at any section between AC at a distance x from A is given by $R_A \times x - M_A$

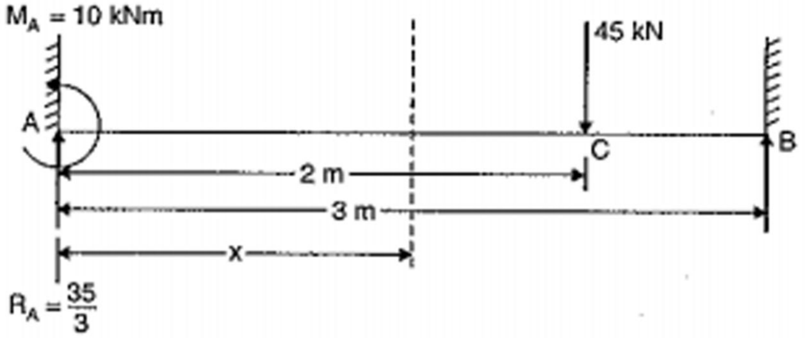


Fig.

or

$$EI\frac{d^2y}{dx^2} = R_A \times x - M_A$$

$$= \frac{35}{3} \times x - 10$$

Integrating, we get

$$EI\frac{dy}{dx} = \frac{35}{3} \times \frac{x^2}{2} - 10x + C_1$$

at $x = 0$, $\frac{dy}{dx} = 0$ $\therefore$ $C_1 = 0$

$$\therefore \quad EI\frac{dy}{dx} = \frac{35}{6}x^2 - 10x \qquad \text{...(iii)}$$

Integrating again, we get

$$EI \times y = \frac{35}{6} \times \frac{x^3}{3} - \frac{10x^2}{2} + C_2$$

at $x = 0, y = 0$, $\therefore$ $C_2 = 0$

$$\therefore \quad EI \times y = \frac{35}{18}x^3 - 5x^2 \qquad \text{...(iv)}$$

(ii) Deflection under the load

From equation (*iv*), we have

$$y = \frac{1}{EI}\left[\frac{35}{18}x^3 - 5x^2\right]$$

To find the deflection under the load, substitute $x = 2$ m in the above equation.

$$\therefore \quad y = \frac{1}{EI}\left[\frac{35}{18} \times 2^3 - 5 \times 2^2\right]$$

$$= \frac{1}{1 \times 10^4}\left[\frac{35 \times 8}{18} - 20\right] \quad (\because \; EI = 1 \times 10^4)$$

$$= -0.000444 \text{ m} = \mathbf{-0.444 \text{ mm}}. \quad \textbf{Ans.}$$

(– ve sign means the deflection is downwards).

(*iii*) *Maximum deflection*

Deflection (y) will be maximum when $\frac{dy}{dx} = 0$.

Hence substituting the value of $\frac{dy}{dx} = 0$ in equation (*iii*), we get

$$0 = \frac{35}{6}x^2 - 10x$$

or $$0 = 35x^2 - 60x$$

or $$0 = x(35x - 60)$$

This means that either $x = 0$ or $35x - 60 = 0$ for maximum deflection.

But x cannot be zero, because when $x = 0$, $y = 0$.

$$\therefore \quad 35x - 60 = 0$$

or $$x = \frac{60}{35} = \frac{12}{7} = 1.714 \text{ m}$$

Substituting $x = 1.714$ m in equation (*iv*), we get maximum deflection.

$$\therefore \quad EIy_{max} = \frac{35}{18}(1.714)^3 - 5(1.714)^2$$

or $$y_{max} = \frac{1}{EI}\left[\frac{35}{18}(1.714)^3 - 5(1.714)^2\right]$$

$$= \frac{1}{1 \times 10^4}[9.79 - 14.69]$$

$$= 0.00049 \text{ m} = \mathbf{0.49 \text{ mm}}. \quad \textbf{Ans.}$$

(*iv*) *Position of maximum deflection*

The maximum deflection will be at a distance of 1.714 m (*i.e.*, $x = 1.714$ m) from end *A*.

Ans.

UNIT : IV

Question : 4 *The shearing stress is a solid shaft is not to exceed 40 N/mm² when the torque transmitted is 20000 N-m. Determine the minimum diameter of the shaft.*

Sol. Given :

Maximum shear stress, $\tau = 40$ N/mm^2

Torque transmitted, $T = 20000$ N-m $= 20000 \times 10^3$ N-mm

Let D = Minimum diameter of the shaft in mm.

Using equation

$$T = \frac{\pi}{16} \tau D^3$$

or $$D = \left(\frac{16T}{\pi\tau}\right)^{1/3} = \left(\frac{16 \times 20000 \times 10^3}{\pi \times 40}\right)^{1/3} = \mathbf{136.2\ mm.} \quad \textbf{Ans.}$$

UNIT : V

Question : 5 *A thick spherical shell of 200 mm internal diameter is subjected to an internal fluid pressure of 7 N/mm². If the permissible tensile stress in the shell material is 8 N/mm², find the thickness of the shell.*

Sol. Given :

Internal dia. $= 200$ mm

∴ Internal radius, $r_1 = 100$ mm

Internal fluid pressure, $p = 7$ N/mm²

Permissible tensile stress, $\sigma_x = 8$ N/mm².

The radial pressure and hoop stress at any radius of spherical shell are given by

$$p_x = \frac{2b}{x^3} - a \quad ...(i) \qquad \text{and} \qquad \sigma_x = \frac{b}{x^3} + a \quad ...(ii)$$

The hoop stress, σ_x will be maximum at the internal radius. Hence permissible tensile stress of 8 N/mm³ is the hoop stress at the internal radius.

At $x = 100$ mm, $p_x = 7$ N/mm².

Substituting these values in equation (*i*), we get

$$7 = \frac{2b}{100^3} - a = \frac{2b}{1000000} - a \quad ...(iii)$$

At $x = 100$ mm, $\sigma_x = 8$ N/mm².

Substituting these values in equation (*ii*), we get

$$8 = \frac{b}{100^3} + a = \frac{b}{1000000} + a \quad ...(iv)$$

Adding equations (*iii*) and (*iv*), we get

$$15 = \frac{3b}{1000000}$$

or $$b = \frac{1000000 \times 15}{3} = 5000000.$$

Substituting the value of b in equation (*iv*), we get

$$8 = \frac{5000000}{1000000} + a = 5 + a$$

∴ $$a = 8 - 5 = 3$$

Substituting the values of a and b in equation (*i*), we get

$$p_x = \frac{2 \times 5000000}{x^3} - 3 \quad ...(v)$$

Let r_2 = External radius of the shell.

At outside, the pressure

$$p_x = 0 \text{ or at } x = r_2, p_x = 0.$$

Substituting these values in equation (*v*), we get

$$0 = \frac{2 \times 5000000}{r_1^3} - 3 \quad \text{or} \quad r_1^3 = \frac{10000000}{3}$$

$$\therefore \qquad r_1 = \left(\frac{10^7}{3}\right)^{1/3} = (3.333)^{1/3} \times 10^2 = 149.3 \text{ mm}$$

∴ Thickness of the shell,

$$t = r_2 - r_1 = 149.3 - 100 = \mathbf{49.3\ mm.} \quad \textbf{Ans.}$$