3CE1A

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3CE1A

B. Tech. (Sem. III) MID TERM Examination,
Civil Engg.
3CE1 Strength of Materials -I

Time : 2 Hours] [Total Marks : 20

UNIT : I

Question : 1 *A tensile test was conducted on a mild steel bar. The following data was obtained from the test :*

(i) Diameter of the steel bar	*= 3 cm*
(ii) Gauge length of the bar	*= 20 cm*
(iii) Load at elastic limit	*= 250 kN*
(iv) Extension at a load of 150 kN	*= 0.21 mm*
(v) Maximum load	*= 380 kN*
(vi) Total extension	*= 60 mm*
(vii) Diameter of the rod at the failure	*= 2.25 cm.*

Determine :(a) the Young's modulus, (b) the stress at elastic limit,
(c) the percentage elongation, and (d) the percentage decrease in area.

Sol. Area of the rod, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (3)^2 \text{ cm}^2$

$$= 7.0685 \text{ cm}^2 = 7.0685 \times 10^{-4} \text{ m}^2. \qquad \left[\because \text{ cm}^2 = \left(\frac{1}{100} \text{ m}\right)^2\right]$$

(*a*) To find Young's modulus, first calculate the value of stress and strain within elastic limit. The load at elastic limit is given but the extension corresponding to the load at elastic limit is not given. But a load of 150 kN (which is within elastic limit) and corresponding extension of 0.21 mm are given. Hence these values are used for stress and strain within elastic limit

$$\therefore \qquad \text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{150 \times 1000}{7.0685 \times 10^{-4}} \text{ N/m}^2 \qquad (\because 1 \text{ kN} = 1000 \text{ N})$$

$$= 21220.9 \times 10^4 \text{ N/m}^2$$

and

$$\text{Strain} = \frac{\text{Increase in length (or Extension)}}{\text{Original length (or Gauge length)}}$$

$$= \frac{0.21 \text{ mm}}{20 \times 10 \text{ mm}} = 0.00105$$

$\therefore$ Young's Modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{21220.9 \times 10^4}{0.00105} = 20209523 \times 10^4 \text{ N/m}^2$$

$$= 202.095 \times 10^9 \text{ N/m}^2 \qquad (\because \quad 10^9 = \text{Giga} = \text{G})$$

$$= \mathbf{202.095 \text{ GN/m}^2}. \quad \textbf{Ans.}$$

(*b*) The stress at the elastic limit is given by,

$$\text{Stress} = \frac{\text{Load at elastic limit}}{\text{Area}} = \frac{250 \times 1000}{7.0685 \times 10^{-4}}$$

$$= 35368 \times 10^4 \text{ N/m}^2$$

$$= 353.68 \times 10^6 \text{ N/m}^2 \qquad (\because \quad 10^6 = \text{Mega} = \text{M})$$

$$= \mathbf{353.68 \text{ MN/m}^2}. \quad \textbf{Ans.}$$

(*c*) The percentage elongation is obtained as,

Percentage elongation

$$= \frac{\text{Total increase in length}}{\text{Original length (or Gauge length)}} \times 100$$

$$= \frac{60 \text{ mm}}{20 \times 10 \text{ mm}} \times 100 = \mathbf{30\%}. \quad \textbf{Ans.}$$

(*d*) The percentage decrease in area is obtained as,

Percentage decrease in area

$$= \frac{(\text{Original area} - \text{Area at the failure})}{\text{Original area}} \times 100$$

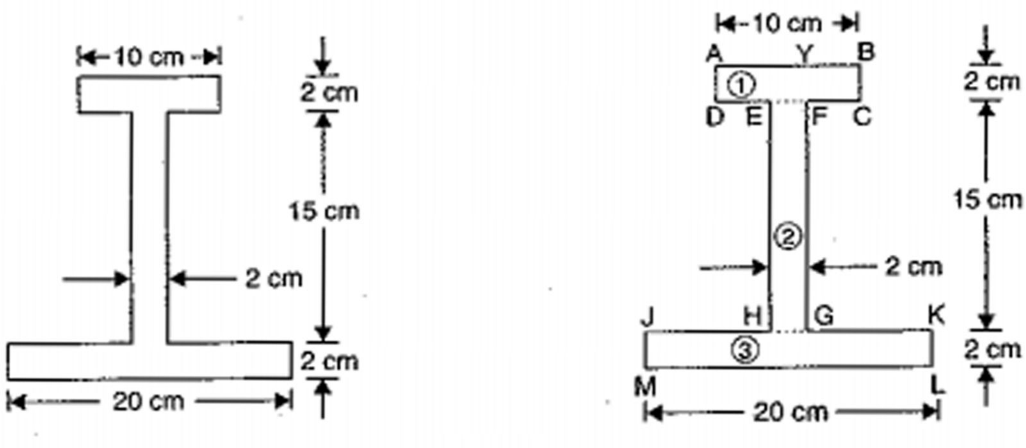
$$= \frac{\left(\frac{\pi}{4} \times 3^2 - \frac{\pi}{4} \times 2.25^2\right)}{\frac{\pi}{4} \times 3^2} \times 100$$

$$= \left(\frac{3^2 - 2.25^2}{3^2}\right) \times 100 = \frac{(9 - 5.0625)}{9} \times 100 = \mathbf{43.75\%}. \quad \textbf{Ans.}$$

UNIT : II

Question : 2 *Find the centre of gravity of the I-section shown in Fig. (a).*

Sol. The *I*-section is split up into three rectangles *ABCD*, *EFGH* and *JKLM* as shown in Fig. (*b*). The given *I*-section is symmetrical about *Y-Y* axis. Hence the C.G. of the section will lie on this axis. The lowest line of the figure line is *ML*. Hence the moment of areas are taken about this line, which is the axis of reference.



(*a*) Fig. (*b*)

Let $\bar{y}$ = Distance of the C.G. of the *I*-section from the bottom line *ML*

a_1 = Area of rectangle *ABCD* = $10 \times 2 = 20\ cm^2$

y_1 = Distance of C.G. of rectangle *ABCD* from bottom line *ML* $= 2 + 15 + \frac{2}{2} = 18$ cm

a_2 = Area of rectangle *EFGH* = $15 \times 2 = 30\ cm^2$

y_2 = Distance of C.G. of rectangle *EFGH* from bottom line *ML* $= 2 + \frac{15}{2} = 2 + 7.5 = 9.5$ cm

a_3 = Area of rectangle *JKLM* = $20 \times 2 = 40\ cm^2$

y_3 = Distance of C.G. of rectangle *JKLM* from bottom line *ML* $= \frac{2}{2} = 1.0$ cm

Now using equation we have $\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{A}$

$$= \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} \qquad (\because\ A = a_1 + a_2 + a_3)$$

$$= \frac{20 \times 18 + 30 \times 9.5 + 40 \times 1}{20 + 30 + 40}$$

$$= \frac{360 + 285 + 40}{90} = \frac{685}{90}$$

= **7.611 cm. Ans.**

UNIT : III

Question : 3 *A hollow alloy tube 5 m long with external and internal diameters 40 mm and 25 mm respectively was found to extend 6.4 mm under a tensile load of 60 kN. Find the buckling load for the tube when used as a column with both ends pinned. Also find the safe load for the tube, taking a factor of safety = 4.*

Sol. Given :

Actual length, $L = 5$ m = 5000 mm

External dia., $D = 40$ mm

Internal dia., $d = 25$ mm

Extension, $\delta L = 6.4$ mm

Tensile load, $W = 60$ kN = 60,000 N

Safety factor = 4

Cross-section area, $A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(40^2 - 25^2) = 766\ mm^2$

Moment of inertia, $I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(40^4 - 25^4)$

$$= \frac{\pi}{64}(2560000 - 390625) = \frac{\pi}{64} \times 2169375 = 106500\ mm^4$$

The value of Young's modulus is obtained as given below

$$E = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{\left(\frac{W}{A}\right)}{\left(\frac{\delta L}{L}\right)}$$

$$= \frac{\left(\frac{60,000}{766}\right)}{\left(\frac{6.4}{5000}\right)} = \frac{60,000}{766} \times \frac{5000}{64} = 6.11945 \times 10^4 \text{ N/mm}^2$$

Since the column is pinned at both the ends.

∴ Effective length, L_e = Actual length = 5000 mm

Let P = Buckling load

Using equation

$$P = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 6.11945 \times 10^4 \times 106500}{5000^2} = \mathbf{2573 \text{ N.}} \quad \textbf{Ans.}$$

And safe load $= \frac{\text{Buckling load}}{\text{Factor of safety}} = \frac{2573}{4} = \mathbf{643.2 \text{ N.}} \quad \textbf{Ans.}$

UNIT : IV

Question : 4 *A horizontal beam AB is simply supported at A and B, 6 m apart. The beam is subjected to a clockwise couple of 300 kNm at a distance of 4 m from the left end as shown in Fig. If $E = 2 \times 10^5$ N/mm² and $I = 2 \times 10^8$ mm⁴, determine :*

(i) deflection at the point where couple is acting and

(ii) the maximum deflection.

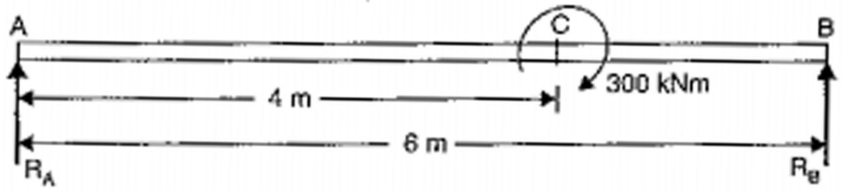


Fig.

Sol. Given :

Length, $L = 6$ m

Couple $= 300$ kNm

Value of $E = 2 \times 10^5$ N/mm²

Value of $I = 2 \times 10^8$ mm⁴

First calculate the reactions R_A and R_B.

Taking moments about A, we get

$$R_B \times 6 = 300$$

∴ $$R_B = \frac{300}{6} = 50 \text{ kN } (\uparrow)$$

and R_A = Total load − R_B = 0 − 50 kN (∵ There is no load on beam)

$= -50$ kN

Negative sign shows that R_A is acting downwards as shown in Fig.

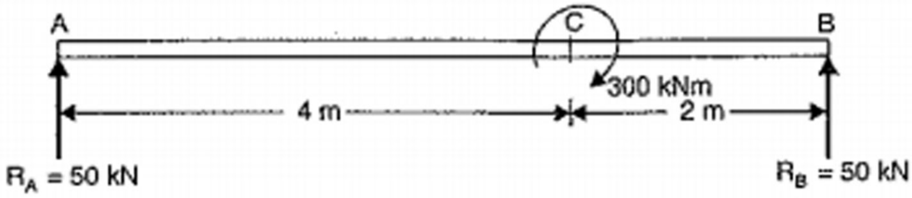


Fig.

The B.M. at any section at a distance x from A, is given by

$$EI\frac{d^2y}{dx^2} = -50x \quad \vdots \quad + 300$$

$$= -50x \quad \vdots \quad + 300(x-4)^0$$

Integrating the above equation, we get

$$EI\frac{dy}{dx} = \frac{-50x^2}{2} + C_1 \quad \vdots \quad + 300(x-4) \qquad ...(i)$$

Integrating again, we get

$$EIy = -\frac{50}{2}\times\frac{x^3}{3} + C_1x + C_2 \quad \vdots \quad + \frac{300\,(x-4)^2}{2}$$

$$= -\frac{25}{3}x^3 + C_1x + C_2 \quad \vdots \quad + 150(x-4)^2 \qquad ...(ii)$$

where C_1 and C_2 are constants of integration. Their values are obtained from boundary conditions which are :

(*i*) at $x = 0, y = 0$ and (*ii*) at $x = 6$ m and $y = 0$.

(*i*) Substituting $x = 0$ and $y = 0$ in equation (*ii*) upto dotted line, we get

$$0 = 0 + C_1 \times 0 + C_2 \quad \therefore \quad C_2 = 0$$

(*ii*) Substituting $x = 6$ m and $y = 0$ in complete equation (*ii*), we get

$$0 = -\frac{25}{3}\times 6^3 + C_1 \times 6 + 0 + 150(6-4)^2$$

$$= -1800 + 6C_1 + 600$$

$$\therefore \qquad C_1 = \frac{1800 - 600}{6} = 200$$

Substituting the values of C_1 and C_2 in equation (*ii*), we get

$$EIy = -\frac{25}{3}x^3 + 200x \quad \vdots \quad + 150(x-4)^2 \qquad (\because C_2 = 0) \quad ...(iii)$$

(*i*) *Deflection at C (i.e., y_C)*

By substituting $x = 4$ in equation (*iii*) upto dotted line, we get the deflection at C.

$$\therefore \qquad EI\,y_C = -\frac{25}{3}\times 4^3 + 200 \times 4$$

$$= -533.33 + 800 = +266.67 \text{ kNm}^3$$

$$= 266.67 \times 10^{12} \text{ Nmm}^3$$

$$\therefore \qquad y_C = \frac{266.67 \times 10^{12}}{2\times 10^5 \times 2 \times 10^8} = \textbf{6.66 mm upwards.} \quad \textbf{Ans.}$$

(ii) Maximum deflection

First find the point where maximum deflection takes place. The maximum deflection is likely to occur in the larger segment AC of the beam. For maximum deflection $\frac{dy}{dx}$ should be zero. Hence equating the slope given by equation (*i*) upto dotted line to zero, we get

$$-\frac{50}{2}x^2 + 200 = 0 \qquad (\because \; C_1 = 200)$$

or

$$-25x^2 + 200 = 0$$

or

$$x = \sqrt{\frac{200}{25}} = 2 \times \sqrt{2} \text{ m}$$

Now substituting $x = 2 \times \sqrt{2}$ in equation (*iii*) upto dotted line, we get the maximum deflection.

$$\therefore \quad EI \times y_{max} = -\frac{25}{3} \times (2 \times \sqrt{2})^3 + 200(2 \times \sqrt{2})$$

$$= -188.56 + 565.68$$

$$= 377.12 \text{ kNm}^3 = 377.12 \times 10^{12} \text{ Nmm}^3$$

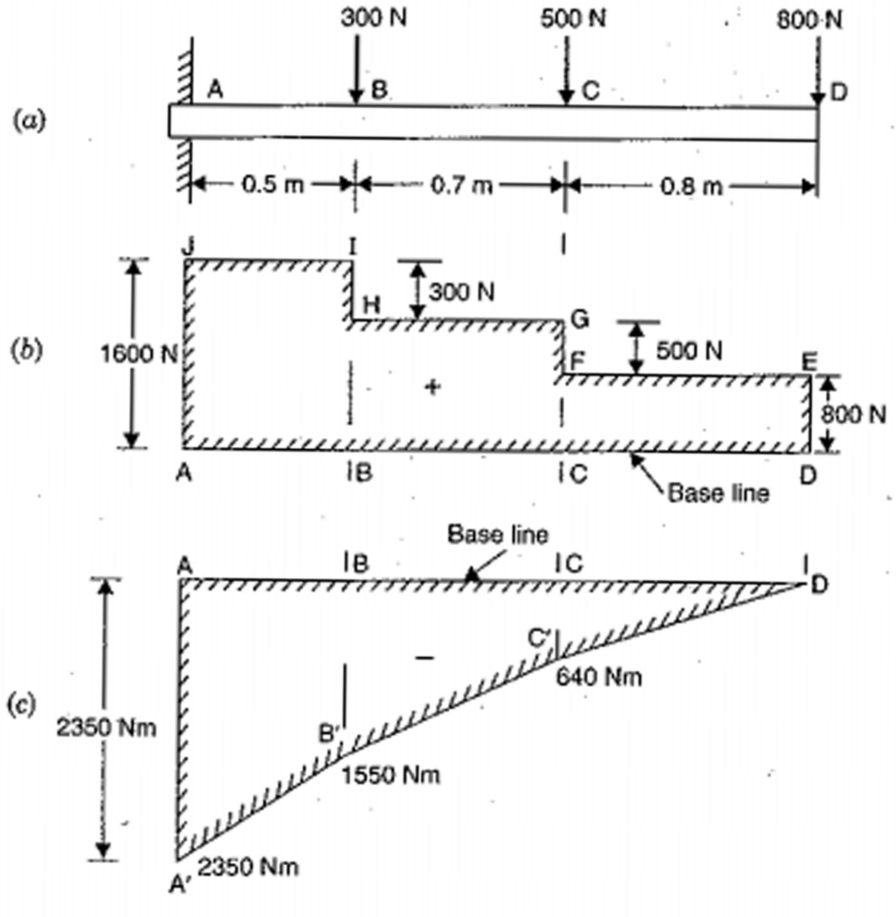
$$\therefore \quad y_{max} = \frac{377.12 \times 10^{12}}{2 \times 10^5 \times 2 \times 10^8} = \textbf{9.428 mm upwards.} \quad \textbf{Ans.}$$

UNIT : V

Question : 5 *A cantilever beam of length 2 m carries the point loads as shown in Fig. Draw the shear force and B.M. diagrams for the cantilever beam.*

Sol. Given :

Refer to Fig.



Shear Force Diagram

The shear force at D is + 800 N. This shear force remains constant between D and C. At C, due to point load, the shear force becomes (800 + 500) = 1300 N. Between C and B, the shear force remains 1300 N. At B again, the shear force becomes (1300 + 300) = 1600 N. The shear force between B and A remains constant and equal to 1600 N. Hence the shear force at different points will be as given below :

S.F. at D, $F_D = +800$ N

S.F. at C, $F_C = +800 + 500 = +1300$ N

S.F. at B, $F_B = +800 + 500 + 300 = 1600$ N

S.F. at A, $F_A = +1600$ N.

The shear force, diagram is shown in Fig. which is drawn as :

Draw a horizontal line AD as base line. On the base line mark the points B and C below the point loads. Take the ordinate $DE = 800$ N in the upward direction. Draw a line EF parallel to AD. The point F is vertically above C. Take vertical line $FG = 500$ N. Through G, draw a horizontal line GH in which point H is vertically above B. Draw vertical line $HI = 300$ N. From I, draw a horizontal line IJ. The point J is vertically above A. This completes the shear force diagram.

Bending Moment Diagram

The bending moment at D is zero :

(*i*) The bending moment at any section between C and D at a distance x and D is given by,

$$M_x = -800 \times x \text{ which follows a straight line law.}$$

At C, the value of $x = 0.8$ m.

$\therefore$ B.M. at C, $M_C = -800 \times 0.8 = -640$ Nm.

(*ii*) The B.M. at any section between B and C at a distance x from D is given by (At C, $x = 0.8$ and at B, $x = 0.8 + 0.7 = 1.5$ m. Hence here x varies from 0.8 to 1.5).

$$M_x = -800x - 500(x - 0.8) \quad \text{...(i)}$$

Bending moment between B and C also varies by a straight line law.

B.M. at B is obtained by substituting $x = 1.5$ m in equation (*i*),

$$\therefore \quad M_B = -800 \times 1.5 - 500(1.5 - 0.8)$$
$$= -1200 - 350 = -1550 \text{ Nm.}$$

(*iii*) The B.M. at any section between A and B at a distance x from D is given by (At B, $x = 1.5$ and at A, $x = 2.0$ m. Hence here x varies from 1.5 m to 2.0 m)

$$M_x = -800x - 500(x - 0.8) - 300(x - 1.5) \quad \text{...(ii)}$$

Bending moment between A and B varies by a straight line law.

B.M. at A is obtained by substituting $x = 2.0$ m in equation (*ii*),

$$\therefore \quad M_A = -800 \times 2 - 500(2 - 0.8) - 300(2 - 1.5)$$
$$= -800 \times 2 - 500 \times 1.2 - 300 \times 0.5$$
$$= -1600 - 600 - 150 = -2350 \text{ Nm.}$$

Hence the bending moments at different points will be as given below :

$$M_D = 0$$
$$M_C = -640 \text{ Nm}$$
$$M_B = -1550 \text{ Nm}$$
and $$M_A = -2350 \text{ Nm.}$$

The bending moment diagram is shown in Fig. which is drawn as.

Draw a horizontal line AD as a base line and mark the points B and C on this line. Take vertical lines $CC' = 640$ Nm, $BB' = 1550$ Nm and $AA' = 2350$ Nm in the downward direction. Join points D, C', B' and A' by straight lines. This completes the bending moment diagram.