

Time : 2 Hours]

[Total Marks : 20



**Problem 1.** (a) Determine the moment of resistance of the rectangular beam of Fig. 13.35.1 having b = 350 mm, d = 600 mm, D = 650 mm,  $A_{st} = 804 \text{ mm}^2$  (4-16T),  $\sigma_{cbc} = 7 \text{ N/mm}^2$  and  $\sigma_{st} = 230 \text{ N/mm}^2$ . (b) Determine the balanced moment of resistance of the beam and the balanced area of tension steel. (c) Determine the actual compressive stress of concrete  $f_{cbc}$  and tensile stress of steel  $f_{st}$  when 60 kNm is applied on the beam. Use direct computation method for all three parts. **Solution 1.** 

**1 (a):** Given data are: b = 350 mm, d = 600 mm,  $A_{st} = 804$  mm<sup>2</sup>,  $\sigma_{cbc} = 7$  N/mm<sup>2</sup> and  $\sigma_{st} = 230$  N/mm<sup>2</sup>.

We have:  $p_t = A_{st}(100) / bd = 80400 / (350) (600) = 0.383$  per cent and  $m = 93.33 / \sigma_{cbc} = 93.33$ /7 = 13.33. We determine the value of *k* from Eq..

 $k = -(p_t m / 100) + \{(p_t m / 100)^2 + (p_t m / 50)\}^{1/2}$ = -0.051 + 0.323 = 0.272 j = 1 - k/3 = 0.909

Equation gives the moment of resistance of the beam as,

$$M = (p_t/100) \sigma_{st} (1 - k/3) bd$$

= (0.383/100) (230) (0.909) (350) (600) (600) = 100.89 kNm.

**1 (b):** The balanced moment of resistances and  $p_{t, bal}$  of the beam is obtained from Eqs.,

 $M_{b} = R_{b}bd^{2}$   $R_{b} = (1/2) \sigma_{cbc} k_{b} j_{b} = (p_{t, bal}/100) \sigma_{st} j_{b}$   $p_{t, bal} = 50 k_{b} (\sigma_{cbc}/\sigma_{st})$ where  $k_{b} = 93.33 / (\sigma_{st} + 93.33)$ 

$$i_{b} = 1 - k_{b}/3$$

 $J_b = 1 - \kappa_b'$ . The values of  $R_b$  and  $p_{t, bal}$  may also be taken from Tables, respectively. Here,  $k_{j} = 93.33 / (230 + 93.33) = 0.288$  and  $j_{j} = 1 - 0.288/3 = 0.904$ .  $p_{t, bal} = 50 \ k_b (\sigma_{cbc} / \sigma_{st}) = 50 \ (0.288) \ (7/230) = 0.438$  $p_{t, bal}$  is 0.44 from Table 13.4.

Therefore,  $M_{b} = (1/2)\sigma_{cbc} k_{b} j_{b} (bd^{2}) = 114.81 \text{ kNm},$ and  $M_{b} = (p_{t, bal}/100) \sigma_{st} j_{b} (bd^{2}) = 114.75 \text{ kNm}$ Taking  $R_{b}$  from Table 13.3, we get:  $M_{b} = R_{b}(bd^{2}) = 0.91 (350) (600)^{2} = 114.66 \text{ kNm}$ 

The balanced area of steel =  $p_{t,bal}(bd/100) = 0.438 (350) (600) / 100 = 919.8 \text{ mm}^2$ . The beam has 4-16 mm diameter bars having 804 mm<sup> $^2$ </sup>. So, additional amount of steel = 919.8 – 804 = 115.8 mm<sup>2</sup> is needed to make the beam balanced.

1'(c): The actual stresses in steel and concrete  $f_{st}$  and  $f_{cbc}$  are obtained from Eqs., respectively.

$$f_{st} = M / \{A_{st} d (1 - k/3)\}$$
  
$$f_{cbc} = (2 A_{st} f_{st}) / (b k d)$$

where  $A_{st} = 804 \text{ mm}^2$  and k = 0.272 (obtained in part a of the solution of this problem). Thus, we have:

 $f_{st} = 60000000 / 804 (600) (0.909) = 136.83 \text{ N/mm}^2 (230 \text{ N/mm}^2)$  $f_{cbc} = 2(804) (136.83) / (350) (0.272) (600) = 3.85 \text{ N/mm}^2 (7 \text{ N/mm}^2)$ 



**Problem 2.** Design a singly-reinforced rectangular beam to carry the design moment = 100 kNm using M 25 concrete and Fe 415 grade of steel. Use b = 300 mm and d = 700 mm (Fig. 13.35.2) as preliminary dimensions.

= 700 mm are given.

## Solution.

Steps 1 and 2 are not needed as the preliminary dimension of b = 300 mm and d = 700 mm are given.

**Step 3.** The balanced moment of resistance of the beam =  $M_h = R_h d^2 = (1.11)$  (300) (700) (700) = 163.17 kNm (taking  $R_{b}$  = 1.11 from Table 13.3 for  $\sigma_{cbc}$  = 8.5 N/mm<sup>2</sup> and  $\sigma_{st}$  = 230 N/mm<sup>2</sup>). The balanced area of steel =  $p_{t, bal}$  (bd) / 100 = (0.53) (300) (700) / 100 = 1113 mm<sup>2</sup> (taking  $p_{t, bal}$  = 0.53 per cent from Table 13.4 for  $\sigma_{cbc}$  = 8.5 N/mm<sup>2</sup> and  $\sigma_{st}$  = 230 N/mm<sup>2</sup>). We provide 4-16 mm diameter bars of  $A_{st}$  = 804 mm<sup>2</sup> ( 1113 mm<sup>2</sup> to have  $p_t$  = 804 (100) / (300) (700) = 0.383 per cent. This is more than the minimum tensile steel = 0.85 bd /  $f_y$  = 430.12 mm<sup>2</sup>, as stipulated in cl. 26.5.1.1 of IS 456.

**Step 4.** Now, the section has to be checked for the stresses in concrete and steel,  $f_{cbc}$  and  $f_{st}$  respectively. For this problem  $m = 93.33 / \sigma_{cbc} = 10.98$  and  $p_t = 0.383$  per cent, obtained in Step 3.

Equation gives:  $k = -(p_t m/100) + {(p_t m/100)}^2 + (p_t m/50) + {(p_t m/50)}^{1/2} = 0.251.$ So, j = 1 - k/3 = 1 - 0.251/3 = 0.916.

Equation gives:  $f_{st} = M / \{A_{st} d (1-k/3)\} = 100 (10^6) / (804) (700) (0.916) = 193.98 \text{ N/mm}^2 \langle 230 \text{ N/mm}^2 \rangle$ .

Equation gives:  $f_{cbc} = 2 A_{st} f_{st} / b kd = 2(804) (193.98) / (300) (0.251) (700) = 5.92 \text{ N/mm}^2 ( 8.5 \text{ N/mm}^2)$ 

Therefore, the beam having b = 300 mm, d = 700 mm and  $A_{st} = 4-16$  mm diameter bars is safe.

Problem 3. Write the limitations of the "Working Stress Method".

# Limitations of Working Stress Method

The basis of the analysis by the working stress method is very simple. This method was used for the design of steel and timber structures also. However, due to the limitations of the method, now the limit state methods are being used. The limitations of working stress method are the following:

- i) The assumptions of linear elastic behaviour and control of stresses within specially defined permissible stresses are unrealistic due to several reasons viz., creep, shrinkage and other long term effects, stress concentration and other secondary effects.
- ii) The actual factor of safety is not known in this method of design. The partial safety factors in the limit state method is more realistic than the concept of permissible stresses in the working stress method to have factor of safety in the design.
- iii) Different types of load acting simultaneously have different degrees of uncertainties. This cannot be taken into account in the working stress method.

Accordingly, the working stress method is gradually replaced by the limit state method. The Indian code IS 456 has given working stress method in Annex B to give greater emphasis to limit state design. Moreover, cl. 18.2.1 of IS 456 specifically mentions of using limit state method normally for structures and structural elements. However, cl.18.2.2 recommends the use of working stress method where the limit state method cannot be conveniently adopted. Due to its simplicity in the concept and applications, better structural performance in service state and conservative design, working stress method is still being used for the design of reinforced concrete bridges, water tanks and chimneys. In fact, design of tension structures and liquid retaining structures are not included in IS 456 for the design guidelines in the limit state method of design.

The design of T-beam in flexure, rectangular and T-beams under shear, torsion and other topics of the limit state method, covered in different lessons, are not included adopting working stress method. However, the designs of tension structures and liquid retaining structures are taken up in the next lesson, as these are not included by the limit state method.



#### Problem 4:

Figure 10.25.6 shows a rectangular short reinforced concrete column using M 25 and Fe 415. Analyse the safety of the column when subjected to  $P_{\mu}$  = 1620 kN and  $M_{\mu}$  = 170 kNm.

## Solution:

This is an analysis type of problems. The data given are: b = 300 mm, D = 450 mm, d' = 56 mm,  $A_{sc} = 4021 \text{ mm}^2$  (20 bars of 16 mm diameter),  $f_{ck} = 25 \text{ N/mm}^2$ ,  $f_y = 415 \text{ N/mm}^2$ ,  $P_u = 1620 \text{ kN}$  and  $M_u = 170 \text{ kNm}$ . So, we have d'/D = 56/450 = 0.1244,  $P_u/f_{ck}bD = 0.48$ ,  $M_u/f_{ck}bD^2 = 0.111934$  and  $p/f_{ck} = 0.11914$ .

## Step 1: Selection of design chart

From the given data: d'/D = 0.1244,  $f_y = 415 \text{ N/mm}^2$  and longitudinal steel bars are equally distributed on four sides, the charts selected are 44 (for d'/D = 0.1) and 45 (for d'/D = 0.15). Linear interpolation has to done with the values obtained from these two charts.

## Step 2: Selection of the particular curve

From the given value of  $p/f_{ck} = 0.11914$ , the two curves having  $p/f_{ck} = 0.1$  and 0.12 are selected from both the charts (No. 44 and 45). Here also, linear interpolation has to be done. **Step 3: Assessment of the column** 

In order to assess the column, we select the two given parameters  $p/f_{ck}$  and  $P_u/f_{ck}bd^2$  to determine the third parameter  $Mu/f_{ck}bD^2$  to compare its value with the given parameter  $M_u/f_{ck}bD^2$ . However, the value of  $M_u/f_{ck}bD^2$  is obtained by doing linear interpolation two times: once with respect to  $p/f_{ck}$ and the second time with respect to d'/D. The results are furnished in Table below:

Table : Values of 
$$M_u / f_{ck} b D^2$$
 when  $(P_u / f_{ck} b D^2)_{given} = 0.48$  and  $(p / f_{ck})_{given} = 0.11914$ ; and  $d' / D = 0.1244$ 

SI. No.	p/f <sub>ck</sub>	d'/D		
		0.1	0.15	0.1244
1	0.1	0.1*	0.09**	0.09512***
2	0.12	0.12*	0.107**	0.113656***
3	0.11914	0.1194***	0.10649***	0.1130941***

Thus, the moment capacity of the column is obtained from the final value of  $M_{u \ ck} bD^2 = 0.1130941$  as

 $M_{\mu}$  = (0.1130941)(25)(300)(450)(450) Nmm = 171.762 kNm,

which is higher than the given  $M_u = 170$  kNm. Hence, the column can be subjected to the pair of given  $P_u$  and  $M_u$  as 1620 kN and 170 kNm, respectively.

## Problem 5:

Design a short spiral column subjected to  $P_u = 2100$  kN and  $M_u = 187.5$  kNm using M 25 and Fe 415. The preliminary diameter of the column may be taken as 500 mm.

#### Solution:

# Step 1: Selection of design chart

With the given  $f_v = 415 \text{ N/mm}^2$  and assuming d'/D = 0.1,

# Step 2: Determination of the percentage of longitudinal steel

With the given  $f_{ck} = 25 \text{ N/mm}^2$  and assuming the given D = 500 mm, we have:

$$P_u f_{ck} D^2 = 2100000/25(500)(500) = 0.336$$
, and

$$M_{u} f_{ck} D^{3} = 187.5(10^{\circ})/25(500)(500)(500) = 0.06$$

The particular point A having coordinates of  $P_{u}/f_{ck}D^2 = 0.336$  and  $M_{u}/f_{ck}D^3 = 0.06$  in Chart 56 gives:  $p/f_{ck} = 0.08$ . Hence, p = 0.08(25) = 2 per cent

 $A_{sc} = 0.02(\pi)(500)(500)/4 = 3928.57 \text{ mm}^2$ 

Provide 8-25 mm diameter bars to have  $A_{sc}$  actually provided = 3927 mm<sup>2</sup>. Marginally less amount of steel than required will be checked considering the enhancement of strength for spiral columns as stipulated in cl.39.4 of IS 456.

#### Step 3: Design of transverse reinforcement:



Fig. Spiral column of Problem 2

The diameter of the helical reinforcement is taken as 8 mm (> 25 mm/4). The pitch p of the spiral is determined, which satisfies the stipulation in cl.39.4.1 of IS 456. From, we have the pitch of the spiral p as:

 $p \le 11.1(D_c - s_p \phi) a_{sp} f_y / (D^2 - f)_{2c} D_{ck} \dots$ where,  $D_c = 500 - 40 - 40 = 420$  mm, D = 500 mm,  $f_{ck} = 25$  N/mm<sup>2</sup>,  $f_y = 415$  N/mm<sup>2</sup>,  $s_p \phi = 8$  mm and  $a_{sp} = 50$  mm<sup>2</sup>.

Using the above values in Eq.10.11, we have  $p \le 25.716$  mm. As per cl.26.5.3.2d1, regarding the pitch of spiral: p >/420/6 (= 70 mm), p </25 mm and p </24 mm. So, pitch of the spiral = 25 mm is o.k. Figure presents the cross-section with reinforcing bars of the column.